

Invariant approach to flavour-dependent CP-violating phases in the MSSM

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ABSTRACT: We use a new weak basis invariant approach to classify all the observable phases in any extension of the Standard Model (SM). We apply this formalism to determine the invariant CP phases in a simplified version of the Minimal Supersymmetric SM with only three non-trivial flavour structures. We propose four experimental measures to fix completely all the observable phases in the model. After these phases have been determined from experiment, we are able to make predictions on any other CP-violating observable in the theory, much in the same way as in the Standard Model all CP-violation observables are proportional to the Jarlskog invariant.

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1. Introduction

From the point of view of theory, the origin of flavour and CP violation constitute two of the most urging questions still unanswered in high energy physics. Different theoretical ideas have been proposed to improve our understanding of these problems [1], but new experimental input is urgently required to unravel this complex puzzle. In the interlude between the LEP and LHC colliders, CP violation and Flavour-Changing-Neutral-Current (FCNC) experiments at low energies are now the main field of research. Even after the start of the LHC, the interplay between the information obtained at the LHC and the information from indirect searches will play a fundamental role in the understanding of CP violation and flavour.

In the Standard Model (SM) both problems are deeply related and the only source of both CP violation and flavour lies in the fermionic Yukawa couplings. In a three-generation SM there is only a single CP-odd quantity invariant under redefinitions of the quark basis. This CP-violating quantity has a nice weak basis invariant formulation with the well-known Jarlskog invariant [2, 3, 4, 5, 6]:

$$J_{\text{CP}} = \det \left(-i \left[Y_u Y_u^\dagger, Y_d Y_d^\dagger \right] \right) = \frac{i}{3} \text{Tr} \left(\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger \right]^3 \right) = \\ - 2 \text{Im} \left[\text{Tr} \left((Y_u Y_u^\dagger)(Y_d Y_d^\dagger)(Y_u Y_u^\dagger)^2(Y_d Y_d^\dagger)^2 \right) \right] \quad (1.1)$$

and all CP-violation effects in the SM are associated to this single observable phase. In general, any extension of the SM includes additional sources of CP violation and new flavour structures, which increase the number of observable phases. Supersymmetry is perhaps the most complete and (theory-) motivated extension of the Standard Model, and we expect to

be able to find the supersymmetric particles in the neighbourhood of the electroweak scale. The Minimal Supersymmetric Standard Model (MSSM) is a perfect example of the increase in the number of observable phases in extensions of the SM. The number of parameters in a generic MSSM, including real flavour parameters and CP phases, is of 124 [7], out of which there are 44 physical phases. Most of these phases have received no attention until recently and only two of them, namely the relative phase between the gaugino masses and the global trilinear phase $\varphi_A = \text{Arg}(A^*M)$ with $\text{Arg}(B\mu) = 0$ and the relative phase between the μ -term in the superpotential and the gaugino masses, $\varphi_\mu = \text{Arg}(\mu^*M)$, have received full attention, thanks to their relation with electric dipole moments (EDMs). However, a generic MSSM introduces many additional mixings and phases, and these parameters have important effects in FCNC and CP-violation experiments [8]. In collider experiments, the presence of SUSY phases can also have a measurable impact [10]. Therefore, if SUSY is found either directly or indirectly in near-future experiments, all the SUSY phases will become observable and a classification of these phases and the construction of fermion basis invariants analogous to the Jarlskog invariant become especially important¹.

In this work we develop a complete formalism that generalizes the construction of weak basis invariants to a generic extension of the SM. The construction of invariants under weak basis transformations (WBTs) to study CP violation in the SM and its extensions has been undertaken for a long time [6, 11, 12, 13, 14, 15, 16]. In this work we extend these analyses by introducing an improved formalism to relate these invariants with observables directly measurable at future experiments. This formalism allows us to translate directly the usual Feynman diagrams into weak basis invariants and vice versa. Furthermore, we are able to define a basis of independent weak basis invariants and to express any invariant in terms of this basis. This program was partly developed in [6] and [16] without the necessary connexion to experimental observables. In particular, in [16] a set of weak basis invariants spanning all the observable phases in the quark sector of a general MSSM was constructed, but the connexion to experimental observables and relations between them was not explicitly presented.

The outline of this work is as follows. In section 2 we define weak basis transformations and present our formalism to build weak basis invariants. We apply this formalism to several simple examples in the SM. In section 3 we analyse the quark sector of a simple MSSM with only three flavour structures, the Yukawa matrices and the squark doublet mass matrix taking all other matrices to be universal. In this simple model we show the full power of our formalism. We find a basis of independent invariants and select a set of experimental observables to fix these invariants. Finally we show that any other CP-violating observable in this model is completely fixed in terms of our basis of invariants, masses and moduli of mixing angles. Therefore, we are able to make predictions on any other CP-violating observable in the theory.

¹Weak bases invariants in the context of the MSSM were originally used by Branco and Kostecky to find the necessary and sufficient conditions for CP conservation in [11].

2. Weak basis transformations and rephasing invariance in the SM

The SM lagrangian density \mathcal{L}_{SM} includes the following $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ contributions

$$\begin{aligned} \mathcal{L}_{G+F+H} = & -\frac{1}{4}G_{\mu\nu}^b G_b^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{q}_L^0 i \not{D} q_L^0 \\ & + \bar{u}_R^0 i \not{D} u_R^0 + \bar{d}_R^0 i \not{D} d_R^0 + (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \end{aligned} \quad (2.1)$$

$$\mathcal{L}_{H+F} = -\bar{q}_L^0 Y_u u_R^0 \tilde{\Phi} - \bar{q}_L^0 Y_d d_R^0 \Phi + \text{h.c.}, \quad (2.2)$$

with D_μ the covariant derivative and the fields q_L^0, u_R^0, d_R^0 in an arbitrary basis with non-diagonal Yukawa couplings. In the SM, we have three copies (i.e. generations) of representations of the $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ gauge group with different masses and mixing angles given by \mathcal{L}_{H+F} . However \mathcal{L}_{G+F+H} only depends on the gauge quantum numbers, and it is completely independent of how we label these three copies; it is invariant under $U(3)_L \otimes U(3)_{u_R} \otimes U(3)_{d_R}$ global transformations acting on q_L , u_R and d_R . These global transformations are WBTs. Two equivalent field assignments are related by

$$q_L^0 = \mathbf{W}_L q_L^{0'} \quad ; \quad u_R^0 = \mathbf{W}_R^u u_R^{0'} \quad ; \quad d_R^0 = \mathbf{W}_R^d d_R^{0'}, \quad (2.3)$$

where $\mathbf{W}_L \in U(3)_L$, $\mathbf{W}_R^u \in U(3)_{u_R}$ and $\mathbf{W}_R^d \in U(3)_{d_R}$. While \mathcal{L}_{G+F+H} is explicitly invariant under these $U(3)_L \otimes U(3)_{u_R} \otimes U(3)_{d_R}$ WBTs, \mathcal{L}_{H+F} is not invariant. Under WBTs, \mathcal{L}_{H+F} changes to

$$\mathcal{L}'_{H+F} = -\bar{q}_L^{0'} Y_u u_R^{0'} \tilde{\Phi} - \bar{q}_L^{0'} Y_d d_R^{0'} \Phi + \text{h.c.}, \quad (2.4)$$

where the fields transform as in Eq. (2.3) and the Yukawa couplings are unchanged. However, if we allow these Yukawa couplings to transform under WBTs as

$$Y_u \rightarrow Y_u' = \mathbf{W}_L^\dagger Y_u \mathbf{W}_R^u \quad ; \quad Y_d \rightarrow Y_d' = \mathbf{W}_L^\dagger Y_d \mathbf{W}_R^d, \quad (2.5)$$

then $\mathcal{L}_{H+F} = \mathcal{L}'_{H+F}$ and the full \mathcal{L}_{SM} is invariant under WBTs. Therefore the two theories given by $\{q_L^{0'}, u_R^{0'}, d_R^{0'}, Y_u', Y_d'\}$ and $\{q_L^0, u_R^0, d_R^0, Y_u, Y_d\}$ provide equivalent *physics*. These two theories correspond to two different choices of the weak basis that we use to formulate our theory. Clearly any physical observable should be independent of our choice and thus weak basis invariant. Physical processes involve quarks with definite mass. The mass eigenstates basis is defined by diagonalizing the Yukawa couplings through biunitary transformations²

$$\mathcal{U}_L^{u\dagger} Y_u \mathcal{U}_R^u = \frac{1}{v} M_u \quad ; \quad \mathcal{U}_L^{d\dagger} Y_d \mathcal{U}_R^d = \frac{1}{v} M_d. \quad (2.6)$$

They correspond to the change of basis

$$u_L^0 = \mathcal{U}_L^u u_L \quad d_L^0 = \mathcal{U}_L^d d_L \quad u_R^0 = \mathcal{U}_R^u u_R \quad d_R^0 = \mathcal{U}_R^d d_R, \quad (2.7)$$

² v is the spontaneous symmetry breaking VEV of Φ .

where u_L, u_R, d_L and d_R are the mass eigenstates. Under this diagonalization the Cabibbo–Kobayashi–Maskawa matrix [17], $\mathcal{U}_L^{u\dagger}\mathcal{U}_L^d \equiv V$ appears in the charged-current couplings. Going to the mass basis in the SM corresponds to the following reparametrization of the lagrangian:

$$(Y_u, Y_d) \rightarrow \left(V, \frac{1}{v}M_u, \frac{1}{v}M_d \right).$$

It must be stressed that the transformation Eq. (2.7) *is not* a WBT: u_L and d_L transform independently. Nevertheless this reparametrization, as is well known, is remarkably useful, among other facts because this new set of parameters is WBT-invariant. First we notice that the biunitary transformations diagonalizing the Yukawa matrices also change under a WBT:

$$\mathcal{U}_L^{u'} = \mathbf{W}_L^\dagger \mathcal{U}_L^u \quad \mathcal{U}_L^{d'} = \mathbf{W}_L^\dagger \mathcal{U}_L^d \quad \mathcal{U}_R^{u'} = \mathbf{W}_R^u \mathcal{U}_R^u \quad \mathcal{U}_R^{d'} = \mathbf{W}_R^d \mathcal{U}_R^d \quad (2.8)$$

Then, we have

$$\begin{aligned} V' &= \mathcal{U}_L^{u'\dagger} \mathcal{U}_L^{d'} = \mathcal{U}_L^{u\dagger} \mathbf{W}_L \mathbf{W}_L^\dagger \mathcal{U}_L^d = \mathcal{U}_L^{u\dagger} \mathcal{U}_L^d = V \\ \frac{M'_u}{v} &= \mathcal{U}_L^{u'\dagger} Y_u' \mathcal{U}_R^{u'} = \mathcal{U}_L^{u\dagger} \mathbf{W}_L \mathbf{W}_L^\dagger Y_u \mathbf{W}_R^u \mathbf{W}_R^{u\dagger} \mathcal{U}_R^u = \mathcal{U}_L^{u\dagger} Y_u \mathcal{U}_R^u = \frac{M_u}{v} \\ \frac{M'_d}{v} &= \mathcal{U}_L^{d'\dagger} Y_d' \mathcal{U}_R^{d'} = \mathcal{U}_L^{d\dagger} \mathbf{W}_L \mathbf{W}_L^\dagger Y_d \mathbf{W}_R^d \mathbf{W}_R^{d\dagger} \mathcal{U}_R^d = \mathcal{U}_L^{d\dagger} Y_d \mathcal{U}_R^d = \frac{M_d}{v} \end{aligned} \quad (2.9)$$

and thus they are clearly weak basis invariants. Nevertheless this common parametrization is not fully defined by Eq. (2.7): as in any diagonalization, the phases of the mass eigenstates are not well defined. This freedom can be incorporated to Eq. (2.7) with the following generalization

$$u_L^0 = \mathcal{U}_L^u e^{i\Theta_L^u} u_L \quad d_L^0 = \mathcal{U}_L^d e^{i\Theta_L^d} d_L \quad u_R^0 = \mathcal{U}_R^u e^{i\Theta_R^u} u_R \quad d_R^0 = \mathcal{U}_R^d e^{i\Theta_R^d} d_R \quad (2.10)$$

with $\Theta_L^u, \Theta_R^u, \Theta_L^d, \Theta_R^d$ real, diagonal matrices. It is precisely this freedom that allows choosing the masses real and positive. Even after this choice, we still have some rephasing freedom that explicitly keeps the diagonal elements in the mass matrix real and positive. This corresponds to rephasing the mass eigenstates with $\Theta_L^u = \Theta_R^u \equiv \Theta^u$, $\Theta_L^d = \Theta_R^d \equiv \Theta^d$; mass eigenvalues are then invariant under those (reduced) rephasings that we consider in the rest of this work. Under the above-mentioned rephasing transformations, V is not invariant and it goes to $e^{-i\Theta^u} V e^{i\Theta^d}$, implying $V_{jk} \rightarrow V_{jk} e^{i(\theta_k^d - \theta_j^u)}$. These are the famous rephasings of the CKM matrix that reduce to a single phase the number of physical phases in a three-generation SM. As a by-product we conclude that physical observables must be both WBT-invariant and rephasing-invariant. Notice, for example, that V is WBT-invariant and rephasing-variant, and thus V will necessarily enter observables through rephasing-invariant combinations, as in Eq. (2.12).

In models where all the flavour couplings in the lagrangian are bilinear in the flavoured fields (as in the SM), WBT and rephasing invariance automatically imply that any physical observable can be written in terms of traces of well-behaved products of flavour matrices. If we define

$$H_u \equiv v^2 Y_u Y_u^\dagger \quad , \quad H_d \equiv v^2 Y_d Y_d^\dagger \quad , \quad H_i \xrightarrow{\text{WBT}} H'_i = \mathbf{W}_L^\dagger H_i \mathbf{W}_L \quad , \quad (2.11)$$

it is well known that any physical observable can be written in terms of:

$$\text{Tr} \left((H_u)^a (H_d)^b (H_u)^c (H_d)^d \dots \right). \quad (2.12)$$

We will call these structures weak basis invariants (WBI). Note that the only matrix transforming under $U(3)_{u_R}$ or $U(3)_{d_R}$ is $Y_j^\dagger Y_j$ ($j = u, d$) and therefore, with this matrix, we can only construct the trivial observable

$$\text{Tr} \left((Y_j^\dagger Y_j)^a \right) = \frac{1}{v^{2a}} \text{Tr} \left((M_j)^{2a} \right) = \frac{1}{v^{2a}} \sum_k (m_{jk})^{2a},$$

implying that right-handed rotations are not observable.

For CP violation it is clear that $\text{Im} [\text{Tr} ((H_u)^a (H_d)^b (H_u)^c (H_d)^d \dots)]$ is a genuine CP-violating phase. Obtaining the Jarlskog invariant [2] in the SM is an instructive exercise. As H_j is hermitian, $\text{Im} [\text{Tr} (H_j)] = \text{Im} [\text{Tr} (H_j H_k)] = 0$; the first invariant with an imaginary part different from zero is

$$J = \text{Im} [\text{Tr} (H_u H_d H_u^2 H_d^2)] = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \text{Im} [V_{22} V_{23}^* V_{33} V_{32}^*]. \quad (2.13)$$

Invariants like Eq. (2.12) and its generalization are quite useful to find out the necessary and sufficient conditions to have CP violation in a given model and therefore to find out the number of independent CP-violating phases [15]. Nevertheless its relation with physical observables is far from obvious. In the SM, J only appears in observables “averaged” over all the quarks, as in the case of the CKM contribution to the electric dipole moments (EDMs) of leptons; Eq. (2.13) never appears in its full glory in CP-violating observables of the quark sector. The reason why J does not appear in CP-violating observables of the quark sector is clear. Equation (2.13) encodes all the necessary conditions to have CP violation, but, if we are able to distinguish a b quark from an s quark experimentally, then a given CP-violating observable involving both quarks does not require the presence of the factor $(m_b^2 - m_s^2)$. Therefore there must be a way, much simpler than Eq. (2.13), of writing WBIs directly related to physical observables.

The key point to reach this goal is to write H_j in terms of projection operators over the mass eigenstates, i.e.

$$H_u = \sum_{i=1}^3 m_{u_i}^2 |u_{Li}\rangle \langle u_{Li}| = \sum_{i=1}^3 m_{u_i}^2 \mathbf{P}_i^{u_L}. \quad (2.14)$$

From Eq. (2.7) it is evident that $[\mathbf{P}_i^{u_L}]_{\alpha\beta} = (\mathcal{U}_L^u)_{\alpha i} (\mathcal{U}_L^{u\dagger})_{i\beta}$, that is

$$\mathbf{P}_i^{u_L} = \mathcal{U}_L^u \mathbf{P}_i \mathcal{U}_L^{u\dagger} \quad ; \quad (\mathbf{P}_i)_{jk} = \delta_{ij} \delta_{ik}. \quad (2.15)$$

These projection operators transform under WBTs (see Eq. (2.8)) as H_u . It is worthwhile to mention that, given Y_u , H_u is perfectly defined and so are \mathcal{U}_L^u and $\mathbf{P}_i^{u_L}$. In general we can define the following chiral projectors with well-defined WBT properties:

$$\mathbf{P}_i^{x_L} = \mathcal{U}_L^x \mathbf{P}_i \mathcal{U}_L^{x\dagger} \quad ; \quad \mathbf{P}_i^{x_L} \rightarrow \mathbf{P}_i^{x_L'} = \mathbf{W}_L^\dagger \mathbf{P}_i^{x_L} \mathbf{W}_L, \quad x = u, d. \quad (2.16)$$

The WBIs in Eq. (2.12) are generalized by allowing any substitution $H_u \rightarrow \mathbf{P}_i^{uL}$ and $H_d \rightarrow \mathbf{P}_i^{dL}$. For right-handed projectors $\mathbf{P}_i^{xR} = \mathcal{U}_R^x \mathbf{P}_i \mathcal{U}_R^{x\dagger}$, $x = u, d$, the following relation

$$v^2 Y_x \mathbf{P}_i^{xR} Y_x^\dagger = (\mathcal{U}_L^x M_x \mathcal{U}_R^{x\dagger})(\mathcal{U}_R^x \mathbf{P}_i \mathcal{U}_R^{x\dagger})(\mathcal{U}_R^x M_x \mathcal{U}_L^{x\dagger}) = m_{x_i}^2 \mathbf{P}_i^{xL} \quad x = u, d \quad (2.17)$$

reflects the inobservability of right-handed rotations and reproduces the well known result that the only thing we need to introduce a right-handed field is a mass insertion. Note that once we use a right-handed projector, it is mandatory to have, inside Eq. (2.17), the string $Y_x \mathbf{P}_i^{xR} Y_x^\dagger$. Equation (2.17) allows us to avoid right-handed projectors.

By using projection operators, the most simple WBI we can construct is

$$\text{Tr} \left(\mathbf{P}_i^{uL} \mathbf{P}_j^{dL} \right) = |V_{ij}|^2 \propto \Gamma(d_{Lj} \rightarrow u_{Li} W), \quad (2.18)$$

where the first equality is obtained using Eq. (2.15) and the last proportionality is trivial from the previous result, but can also be obtained from the presence of two projectors, which means the square of the amplitude $\langle u_{Li} | \mathbf{1} | d_{Lj} \rangle$. Note that in the weak basis where we are working, the flavour structure of the W coupling is just the identity. This example shows that, using projection operators, one can write much simpler WBIs, directly related to physical processes. The first kind of CP-violating WBI made with projectors is

$$\begin{aligned} \text{Im} \left[\text{Tr} \left(\mathbf{P}_1^{uL} \mathbf{P}_1^{dL} \mathbf{P}_2^{uL} \mathbf{P}_2^{dL} \right) \right] &= \text{Im} [V_{11} V_{21}^* V_{22} V_{12}^*] \propto \\ &\propto \Gamma(D_s^+ \rightarrow K^0 \pi^+) - \Gamma(D_s^- \rightarrow \bar{K}^0 \pi^-). \end{aligned} \quad (2.19)$$

This WBI is the well celebrated imaginary part of the quartets of the CKM matrix. It must be related to the CP-violating interference of two different weak amplitudes that appear at tree level, because there are no internal masses. This is the case for $D_s^+ \rightarrow K^0 \pi^+$, where the interfering amplitudes are the decay $c \rightarrow u d \bar{d}$ and the annihilation $(c \bar{s} \rightarrow u \bar{s})^*$; it is clear that in this interference we have exactly the four projectors in Eq. (2.19): $u\bar{u}$, $d\bar{d}$, $c\bar{c}$ and $s\bar{s}$. It is worthwhile to mention that, from the experimental side, in this decay we are tagging the quarks u , d , c and s , and there is therefore no need for any mass suppression factor such as the ones in Eq. (2.13)³. Notice that, in addition, non-zero strong phase differences are required. WBIs especially useful are those that involve projectors and the flavour structures of the lagrangian as H_u and H_d . These invariants are discussed in the next section in a wider context.

3. The Minimal Supersymmetric SM

In this section we analyse WBIs in the MSSM. If no interactions connect leptons and quarks, as is the case of an R-parity-conserving MSSM, we can consider both sectors separately. In this work we concentrate on quarks. A general MSSM involves 7 independent flavour matrices in the quark sector [18]. These 7 flavour matrices are:

$$Y_u, \quad Y_d, \quad Y_u^A, \quad Y_d^A, \quad M_Q^2, \quad M_U^2, \quad M_D^2. \quad (3.1)$$

³Note that in the decay $D_s^+ \rightarrow K^0 \pi^+$ there is also a highly suppressed penguin contribution.

In some scenarios, such as the so-called Constrained MSSM [19, 20] or Minimal Flavour Violation models, the soft mass matrices are supposed to be universal and the trilinear matrices proportional to the Yukawa matrices [21]. However, in realistic supersymmetric flavour models [22] we expect all these matrices to have non-trivial flavour structures. Here, as a first approach to this enlarged flavour scenario, we will consider a simplified situation with 3 non-trivial flavour matrices; in this restricted MSSM, Y_u , Y_d , and M_Q^2 are generic matrices, while the remaining matrices formula (3.1) are:

$$\begin{aligned} M_U^2 &= m_u^2 \mathbf{1} & M_D^2 &= m_d^2 \mathbf{1} \\ Y_u^A &= A_0 Y_u = A_0^* Y_u & Y_d^A &= A_0 Y_d = A_0^* Y_d \end{aligned} \quad (3.2)$$

where m_u^2 , m_d^2 and A_0 are real numbers; $H_{\bar{Q}} \equiv M_Q^2$ is hermitian and, under a WBT, transforms as H_u and H_d in Eq. (2.11). Furthermore, as we are mainly interested in flavour-dependent phases we also take a real μ parameter in the superpotential. Using the same strategy as was already used for the SM in the previous section, we can build a complete set of invariants in our simple MSSM model. The number of independent parameters, and thus of independent observables, can be determined as shown in [23]:

$$N = N_{Fl} - N_G + N_{G'}, \quad (3.3)$$

where N_{Fl} is the number of parameters in the flavour matrices, N_G is the number of parameters of the WBTs group $G = U(3)_L \otimes U(3)_{u_R} \otimes U(3)_{d_R}$, and G' is the subgroup of G under which the flavour matrices are invariant, that is the subgroup of G unbroken by the flavour matrices. Equation (3.3) applies separately to mixings+masses and to phases. In our simple MSSM, we have $N_{Fl} = 2 \times 18 + 9 = 45$, $N_G = 3 \times 9 = 27$ and the unbroken subgroup is only $U(1)$ corresponding to baryon number conservation, $N_{G'} = 1$. Therefore this yields 9 masses, 3×2 mixing angles and 4 CP-violating phases. Corresponding to the 4 CP-violating phases in the model we only need 4 independent complex invariants to describe CP violation. Using the formalism developed in the previous section, it is clear that we can build an infinite number of complex invariants. However, as we prove in Appendix A, we can always express any invariant in terms of a chosen set of four independent invariants.

A second ingredient needed to determine all the independent phases in our model is the possibility to relate these independent invariants to physical observables. As we saw in the previous section, this is achieved through the introduction of projectors on external states. In fact, we can make a direct correspondence between these invariants and Feynman-like flavour diagrams. Starting from a (set of) Feynman diagram(s) we can immediately read the corresponding WBI(s) and, conversely, we can draw the Feynman diagrams corresponding to a given invariant. To do this we only need a few considerations:

- Full invariants and physical observables always correspond to cross sections or decay rates and hence squared moduli of amplitudes (Feynman diagrams).
- We obtain a flavour loop by joining a Feynman diagram with a conjugated Feynman diagram contributing to the same amplitude. In theories where the couplings are always bilinear in flavour we obtain a closed *flavour path* corresponding directly to a trace.

- Each initial or final particle is represented by a projector on the corresponding mass eigenstate.
- To every virtual particle in loops we associate a full flavour structure, H_u , H_d or $H_{\tilde{Q}}$. Strictly speaking we must include an arbitrary function of those matrices, which can be expanded: $f(H_X, H_Y \dots) = \sum_{nm} C_{nm} H_X^n H_Y^m \dots$.
- In a given *flavour path* any transition between different flavour matrices (projectors or full flavour structures) is mediated by the appropriate flavour-blind gauge or gaugino lines taking into account the charge and spin of the particles involved: $W^+ \equiv H_u \leftrightarrow H_d$, χ^0 , $\tilde{g} \equiv H_u \leftrightarrow H_{\tilde{Q}}(u)$, $\chi^+ \equiv H_u \leftrightarrow H_{\tilde{Q}}(d)$, \dots .
- Neutral gauge bosons do not modify flavour, hence an arbitrary number of them may be attached at any point of our *flavour path*.

Using these as rules, it is straightforward to translate invariants into Feynman diagrams and vice versa. However, it is important to notice that a given invariant may correspond to different processes. For instance adding any number of external photons, gluons or Z bosons to our Feynman diagram does not modify the corresponding invariant.

To illustrate the use of the method exposed in this work we analyse a simple observable, the CP-violating asymmetry in $Z \rightarrow \bar{b}s$ and $Z \rightarrow b\bar{s}$ decays. Although the example is fully developed in Appendix B, we show in Fig. 1 (drawn using Jaxodraw [24]) one of the contributions to the asymmetry: the interference between the standard amplitude of Fig. 1(a) and the new amplitude of Fig. 1(b), closing a *flavour path* and joining both diagrams. The closed *flavour path* shadowed in Fig. 1 gives the invariant trace (reading clockwise): $\text{Tr} \left(\mathbf{P}_2^{dL} F(H_u) \mathbf{P}_3^{dL} G^\dagger(H_{\tilde{Q}}) \right)$; as explained in Appendix B, $F(H_u)$ and $G(H_{\tilde{Q}})$ are loop functions and the circles ‘o’ over the s and b quark lines correspond to the flavour projectors \mathbf{P}_2^{dL} and \mathbf{P}_3^{dL} .

At this point we already have the necessary tools to choose our basis of 4 independent invariants that fix all the observable phases in our MSSM model and to relate them to physical observables. In the first place we notice that we only have to consider invariants built with projector operators. This is because any invariant, including full flavour structures, H_a with $a = u, d, \tilde{Q}$, can always be written as a linear combination of invariants built with projectors and masses⁴:

$$\text{Tr} \left((H_u)^a (H_{\tilde{Q}})^b (H_d)^c \dots \right) = \sum_{\alpha\beta\gamma} (m_{u_\alpha}^2)^a (m_{\tilde{Q}_\beta}^2)^b (m_{d_\gamma}^2)^c \text{Tr} \left(\mathbf{P}_\alpha^{uL} \mathbf{P}_\beta^{Q} \mathbf{P}_\gamma^{dL} \dots \right). \quad (3.4)$$

The first complex invariant we can build involves, at least, three different projectors, $\text{Tr} \left(\mathbf{P}_i^{uL} \mathbf{P}_j^{dL} \mathbf{P}_k^Q \right)$. However, it is more convenient to consider invariants with four matrices, for instance $\text{Tr} \left(\mathbf{P}_i^{uL} \mathbf{P}_j^{dL} \mathbf{P}_k^Q \mathbf{P}_l^{dL} \right)$. In fact, these four projector invariants correspond

⁴Strictly speaking, after electroweak symmetry breaking, left-handed squarks mix with right-handed squarks in a 6×6 mass matrix and the chirality of the eigenvalues is not well defined. Here we can safely neglect the small left–right mixing proportional to Yukawa couplings. This issue will be further addressed in [25].

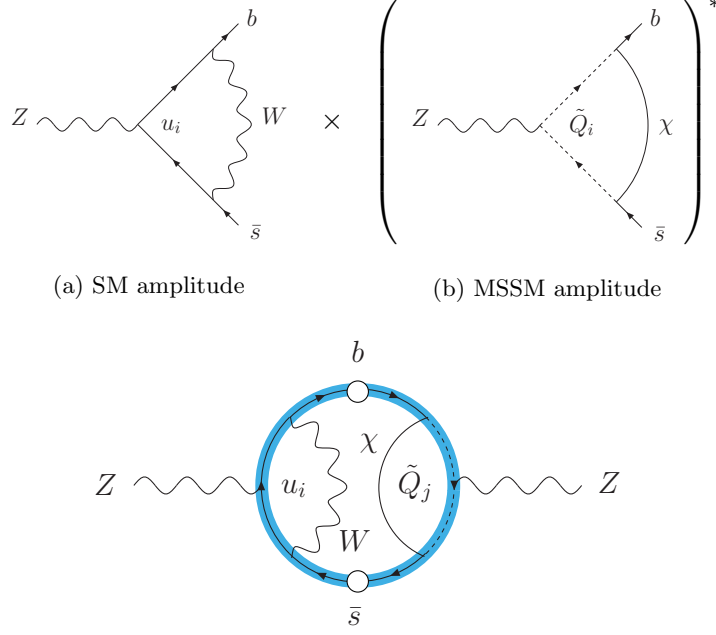


Figure 1: Interference term contributing to $\Gamma(Z \rightarrow b\bar{s}) - \Gamma(Z \rightarrow \bar{b}s)$.

directly to the familiar rephasing invariant quartets of mixing matrices. Writing our hermitian matrices in terms of masses and relative misalignments,

$$H_d = D_d, \quad H_u = V^\dagger D_u V, \quad H_{\tilde{Q}} = U^\dagger D_{\tilde{Q}} U, \quad (3.5)$$

where D_a are diagonal matrices with eigenvalues $m_{a_i}^2$. We have

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_l^{d_L} \right) = V_{ij} U_{kl} V_{il}^* U_{kj}^*.$$

For this reason we will select our independent invariants from the invariants with four projectors. All other complex invariants can be written in terms of four matrices invariants using the techniques in Appendix A. In particular, using $\mathbf{1} = \sum_l P_l^{d_L}$, it is trivial to write invariants of three matrices in terms of four matrices invariants, $\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q \right) = \sum_l \text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_l^{d_L} \right)$.

The 4 projector invariants will involve at least two projectors of the same kind. They can be one of the following structures:

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^{u_L} P_l^{d_L} \right), \quad \text{Tr} \left(P_i^{u_L} P_j^Q P_k^{u_L} P_l^Q \right), \quad \text{Tr} \left(P_i^Q P_j^{d_L} P_k^Q P_l^{d_L} \right) \quad (3.6)$$

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_l^{d_L} \right), \quad \text{Tr} \left(P_i^{u_L} P_j^Q P_k^{d_L} P_l^Q \right), \quad \text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^{u_L} P_l^Q \right). \quad (3.7)$$

As shown in Appendix A, all these different structures can be reduced to three families of

invariants:

$$\begin{aligned}
J_{ij,kl}^{(V)} &\equiv \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^{u_L} \mathbf{P}_l^{d_L} \right) = V_{ij} V_{kl} V_{il}^* V_{kj}^* \\
J_{ij,kl}^{(U)} &\equiv \text{Tr} \left(\mathbf{P}_i^Q \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_l^{d_L} \right) = U_{ij} U_{kl} U_{il}^* U_{kj}^* \\
I_{ij,kl} &\equiv \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_l^{d_L} \right) = V_{ij} U_{kl} V_{il}^* U_{kj}^*.
\end{aligned} \tag{3.8}$$

From here it is clear that $J_{ij,kl}^{(V)}$ are the familiar rephasing invariant quartets of the CKM mixing matrix (notice that V corresponds to the CKM mixing matrix). It is well known that all the quartets we can build have the same imaginary part and therefore we only need one of them plus the moduli of the CKM elements to fix all of them. The same is true for the quartets $J_{ij,kl}^{(U)}$, although this time in terms of the relative misalignment between squark doublets and down quarks. Therefore we choose as independent quartets $J_{32,23}^{(V)}$ and $J_{32,23}^{(U)}$.

There are similar properties relating the different $I_{ij,kl}$ quartets. Using the properties listed in Appendix A, all these $I_{ij,kl}$, $J_{ij,kl}^{(V)}$ and $J_{ij,kl}^{(U)}$ can be written in terms of only four independent quartets, which we choose to be

$$\begin{aligned}
J_{32,23}^{(V)} &\equiv \text{Tr} \left(\mathbf{P}_3^{u_L} \mathbf{P}_2^{d_L} \mathbf{P}_2^{u_L} \mathbf{P}_3^{d_L} \right), & J_{32,23}^{(U)} &\equiv \text{Tr} \left(\mathbf{P}_3^Q \mathbf{P}_2^{d_L} \mathbf{P}_2^Q \mathbf{P}_3^{d_L} \right), \\
I_{33,32} &\equiv \text{Tr} \left(\mathbf{P}_3^{u_L} \mathbf{P}_3^{d_L} \mathbf{P}_3^Q \mathbf{P}_2^{d_L} \right), & I_{32,31} &\equiv \text{Tr} \left(\mathbf{P}_3^{u_L} \mathbf{P}_2^{d_L} \mathbf{P}_3^Q \mathbf{P}_1^{d_L} \right),
\end{aligned} \tag{3.9}$$

plus squared moduli of elements of the mixing matrices. These four invariants constitute a basis of linearly independent complex invariants. Any other complex invariant we can build in this theory can be uniquely expressed as a linear combination of these four invariants with coefficients proportional to masses and moduli of elements of the mixing matrices. This is one of the key results of this work as we are now able to relate unambiguously all the possible CP-violating quantities of the theory and therefore make predictions on different observables.

The last step is to relate these four independent invariants to physical observables where they can be measured. So far supersymmetric particles have not been directly observed and we will probably have to wait until the LHC is in operation before we can analyse processes with SUSY particles as external states. In the meantime, we can use FCNC and CP-violation experiments to measure new contributions with SUSY particles running in the loops. Consequently we choose our four independent observables within this class of processes.

At the moment, CP violation has only been observed in neutral kaon and neutral B systems. These measurements seem to be consistent with a Standard Model interpretation of the observed CP violation [26]. Nevertheless, any extension of the SM predicts some departure from the SM expectations once the experimental and theoretical precision is improved. On the other hand, the CKM Jarlskog quartet is also included in our independent set of invariants and must be determined from the experimental data. Therefore it is convenient to include in our set of observables the two best experimental determinations of CP violation, indirect CP violation in the neutral kaon system – ε_K – and the CP asymmetry in $B^0 \rightarrow J/\psi K_S$.

ε_K corresponds to a particular combination of neutral kaon decay rates: K_L and K_S decay rates with $I = 0$, so that we select CP violation in K^0 – \bar{K}^0 mixing. One contribution to these decays is, for example, the tree-level $K^0 \rightarrow \pi^+\pi^-$ and the mixing-mediated decay $K^0 \rightarrow \bar{K}^0 \rightarrow \pi^+\pi^-$. In this case the external particles are two final-state pions and both the K^0 and the \bar{K}^0 , as we select explicitly CP violation in K^0 – \bar{K}^0 mixing. Therefore we need two P_1^u and two P_1^d projectors corresponding to the pions, also two P_2^d projectors and two P_1^d projectors, the projectors corresponding to the K^0 and \bar{K}^0 . Naturally these processes will have contributions from SM loops and new contributions from the virtual SUSY particles [27, 28]. In Fig. 2 we show one of the contributions to the interference between the tree-level SM amplitude (Fig. 2(a)) and the SUSY-mixing-mediated (Fig. 2(b)) amplitude, that is, the leading contribution beyond the SM. The invariant corresponding to this contribution is easily obtained: starting from an s quark projector, the *flavour path* (shadowed) in Fig. 2 reads: $\left[\text{Tr} \left(P_2^{dL} P_1^{uL} P_1^{dL} H_{\tilde{Q}} \right) \right]^2$. More accurately the exact structure would be an arbitrary function $F(H_{\tilde{Q}}, H_{\tilde{Q}})$, where $F(m_{\tilde{Q}_i}^2, m_{\tilde{Q}_j}^2) = \sum_{m,n} C_{mn} m_{\tilde{Q}_i}^{2m}, m_{\tilde{Q}_j}^{2n}$ will be the corresponding Inami-Lim function [29], see Appendix B for an example. For simplicity, in the following we will only consider the leading term of these expansions.

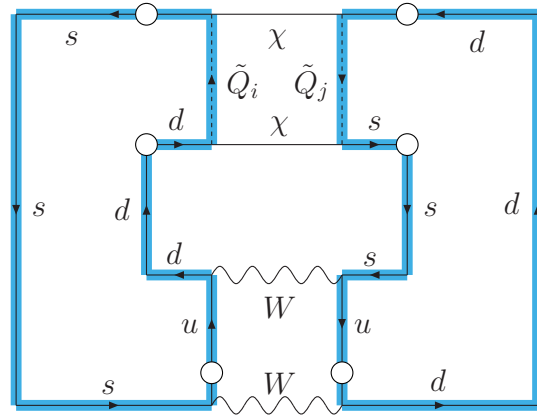
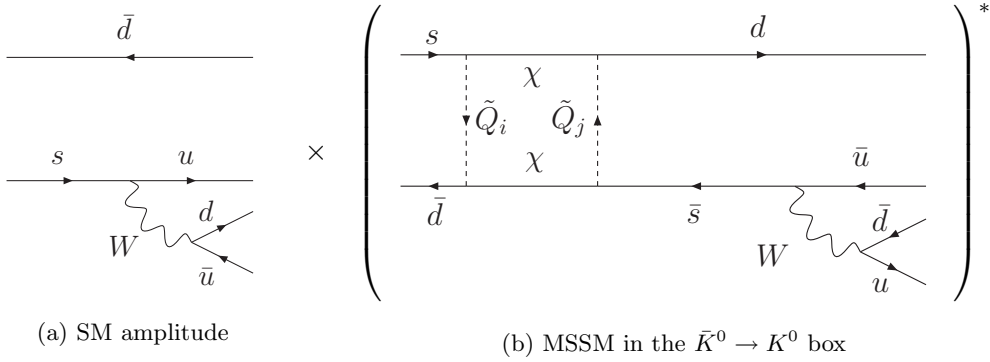


Figure 2: CP violating contribution in $K^0 \rightarrow \pi\pi$.

Other contributions to the interference follow from Fig. 2 by crossing internal lines in

the $K^0-\bar{K}^0$ (box) mixing; they give rise to a second trace: $\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \right)$. As in Appendix A, we easily reduce

$$\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \right) = \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \right) \right]^2.$$

Thus all the interference terms with SUSY particles running in the neutral kaon mixing share a flavour invariant structure. SM contributions involve up quarks and W 's in the mixing, thus giving terms proportional to:

$$\left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_u \right) \right]^2.$$

Consequently ε_K depends on the following complex invariants:

$$\varepsilon_K = C_{\text{SM}}^{\varepsilon_K} \text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_u \right)^2 \right] + C_{\text{MSSM}}^{\varepsilon_K} \text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_1^{uL} \mathbf{P}_1^{dL} H_{\tilde{Q}} \right)^2 \right], \quad (3.10)$$

where $C_{\text{SM}}^{\varepsilon_K}$ and $C_{\text{MSSM}}^{\varepsilon_K}$ are real coefficients that depend on coupling constants and real invariants.

The complex invariants relevant to the description of CP violation in $B^0 \rightarrow J/\psi K_S$ are obtained as in the ε_K case. The only difference is the presence of an additional neutral meson mixing because we have both $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ mixings, implying that the analogue of Fig. 2 involves an additional box contribution. The $B^0 \rightarrow J/\psi K_S$ asymmetry is $A_{\text{CP}}(J/\psi K_S) \propto \sin(2\varphi_{J/\psi K_S})$ where $\varphi_{J/\psi K_S}$ is given by

$$\varphi_{J/\psi K_S} \equiv \arg \left\{ \sum_{i,j=u,\tilde{Q}} C_{ij}^{J/\psi K_S} \left(\text{Tr} \left(\mathbf{P}_1^{dL} H_i \mathbf{P}_3^{dL} \mathbf{P}_2^{uL} \right) \text{Tr} \left(\mathbf{P}_1^{dL} \mathbf{P}_2^{uL} \mathbf{P}_2^{dL} H_j \right) \right)^2 \right\} \quad (3.11)$$

and $C_{ij}^{J/\psi K_S}$ are, again, real coefficients that depend on coupling constants and real invariants. In this case, we expect a large contribution from the SM to this phase. However, a sizeable SUSY contribution proportional to $\text{Tr} \left(\mathbf{P}_1^{dL} H_{\tilde{Q}} \mathbf{P}_3^{dL} \mathbf{P}_2^{uL} \right)$ is still possible [27, 30] and can play a relevant role in the unitarity triangle fit.

The third observable we are going to choose is the CP asymmetry in $B_s \rightarrow J/\psi \Phi$ or $B_s \rightarrow D_s^+ D_s^-$. Notice that both processes correspond exactly to the same decays at the quark level and hence give rise to the same invariant. This channel is especially interesting for several reasons. First, many realizations of supersymmetry can give a sizeable contribution to $B_s-\bar{B}_s$ mixing with a large phase [31]. Then, the SM contribution to the CP asymmetry is very small and therefore a sizeable CP asymmetry would be a signal of new physics. Finally, this asymmetry is accessible at B -physics experiments at hadron colliders such as LHCb or BTeV.

In this case, the diagrams are analogous to the ε_K diagrams shown in Fig. 2. We also have a SM contribution to the mixing and a new contribution from SUSY. The corresponding invariants are $\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_u \right)^2 \right]$ for the SM contribution and $\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_{\tilde{Q}} \right)^2 \right]$ for the MSSM contribution. This CP asymmetry is approximately dominated by the tree-level decay amplitude [32], and therefore $A_{\text{CP}}(B_s \rightarrow D_s^+ D_s^-) \propto \sin(2\varphi_{D_s^+ D_s^-})$. The presence

of a single mixing ($B^0-\bar{B}^0$) simplifies the analogue of Eq. (3.11) and this phase is given by

$$\sin(\varphi_{D_s^+ D_s^-}) = \frac{\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_u \right)^2 \right] + C_S \text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_{\tilde{Q}} \right)^2 \right]}{\left| \text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_u \right)^2 + C_S \text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_{\tilde{Q}} \right)^2 \right|}. \quad (3.12)$$

The coefficient C_S takes into account the differences in (real) couplings and masses from the SM and the new SUSY contributions; it is known from other CP-conserving measurements. As said above, the SM contribution to this asymmetry is small:

$$\varphi_{\text{SM}} = \arg \left\{ \text{Tr} \left(\mathbf{P}_2^{dL} \mathbf{P}_2^{uL} \mathbf{P}_3^{dL} H_u \right)^2 \right\} \simeq \mathcal{O}(\lambda_c^2), \quad (3.13)$$

with λ_c the Cabibbo angle. Thus, in practice, this contribution can be safely neglected in the presence of a sizeable new physics contribution.

Finally, we need a fourth observable to obtain our four independent invariants. The choice now is more difficult, and there is no clear option. However, we choose the CP asymmetry in the $b \rightarrow s\gamma$ decay, which is already being measured at the B factories [33] and corresponds to a new invariant, independent of the invariants involved in the previous observables [34, 20]. Notice that this process entails a change in the chirality of the down quarks, i.e. it is a transition $b_R \rightarrow s_L \gamma$. This implies that we now need a right-handed projector \mathbf{P}_3^{dR} ; however, using Eq. (2.17), we have $Y_d \mathbf{P}_3^{dR} Y_d^\dagger = m_b^2 \mathbf{P}_3^{dL}$. Therefore, with the exception of this additional quark mass, the diagrams involved are completely analogous to the diagrams in the $Z \rightarrow b\bar{s}$ asymmetry and we have,

$$A_{\text{CP}}(b \rightarrow s\gamma) \propto \text{Im} \left[\text{Tr} \left(\mathbf{P}_3^{dL} H_u \mathbf{P}_2^{dL} H_{\tilde{Q}} \right) \right]. \quad (3.14)$$

In summary, in Eqs. (3.10)–(3.14) we have four observables that can be expressed as functions of our four independent invariants using the relations of Appendix A. Therefore we have four equations and four unknowns and we can fix completely the four CP-violating invariants of our MSSM. This implies that any other CP violation observable in this model is already fixed in terms of our four invariants and masses or moduli of mixing angles.

For instance, we can now calculate in our model the CP asymmetry in the $B_d \rightarrow \phi K_S$ decay, which could show a discrepancy from the SM expectations [31]. In this case the relevant invariant, assuming that the large SUSY contribution is in the decay amplitude while the $B-\bar{B}$ mixing is SM-dominated, would be

$$\begin{aligned} \sin(\varphi_{\phi K_S}) &\propto \text{Im} \left[\text{Tr} \left(\mathbf{P}_3^{dL} H_{\tilde{Q}} \mathbf{P}_2^{dL} H_u \mathbf{P}_1^{dL} H_u \right) \right] \\ &\simeq \frac{m_t^2}{|V_{tb}|^2} \text{Im} \left[\text{Tr} \left(\mathbf{P}_3^{dL} H_{\tilde{Q}} \mathbf{P}_2^{dL} \mathbf{P}_3^{uL} \right) \right] \text{Re} \left[\text{Tr} \left(\mathbf{P}_1^{dL} H_u \mathbf{P}_3^{dL} \mathbf{P}_3^{uL} \right) \right]. \end{aligned} \quad (3.15)$$

So, it is clear that this asymmetry in our model is directly related to the CP asymmetry in $b \rightarrow s\gamma$ decays. Note that this is only due to the fact that there are no other Left-Right couplings in our reduced MSSM model, apart from the usual Yukawas. Naturally, in a complete MSSM, this relation may be destroyed by these additional couplings. This

kind of relations can be extended to any other CP-violating observable in the theory, for instance new SUSY contributions to ε'/ε [35], $K \rightarrow \pi\nu\bar{\nu}$ [36], and possibly CP asymmetries at future linear colliders [9, 10].

Let us now briefly discuss the realistic experimental determination of these observables. First, we must emphasize that the situation regarding possible new physics contributions in B-factories has changed dramatically in recent times. Babar [37] and Belle [38] have presented for the first time a measurement of the phase $\gamma = \arg(-V_{ud}V_{ub}^*V_{cd}^*V_{cb})$ from the "tree-level" decays $B^\pm \rightarrow DK^\pm$, $B^\pm \rightarrow D^*K^\pm \rightarrow (D\pi^0)K^\pm$, where the two paths to D^0 or \bar{D}^0 interfere in the common decay channel $\bar{D}^0, D^0 \rightarrow K_S\pi^+\pi^-$. This measurement corresponds to the determination of the pure SM phase in B decays, $\arg[\text{Tr}(P_3^{uL}P_2^{dL}P_2^{uL}P_3^{dL})]$. Even more important: B factories will achieve a measurement of γ with a precision of a few degrees in the near future [39]. Using this observable together with the tree level observables $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ allows a high precision determination of the full V_{CKM} "independent" of the presence of any new physics that respects 3×3 unitarity, as the MSSM analysed here. At this point, any other FCNC and/or CP violating observable could be devoted to the search of new phases as deviations from this tree level measurement. In this scenario, it would be enough to use γ together with ϵ_K , $A_{CP}(b \rightarrow s\gamma)$ and $\varphi_{J/\psi K_S}$. As is well known, at present the first two FCNC observables agree with the SM prediction and, in principle, can only accommodate a relatively small MSSM contribution. Recently [40, 41] a model independent analysis of $\varphi_{J/\psi K_S}$ has shown that there are two solutions for this phase, one of them clearly outside the SM. Nevertheless the statistical significance of this second solution is smaller than the SM one [42], and it is possible that this second solution gets even less significant when more data become available [40, 41, 42]. Thus we prefer to wait for an updated analysis before we can consider this possibility a genuine new physics hint. Now the experimental determination of the CP violating phase in $B_s^0-\bar{B}_s^0$, $\varphi_{D_s^+D_s^-}$ is going to be of paramount importance, specially taking into account its small value in the SM. A clear signal of deviations from the SM in $\varphi_{D_s^+D_s^-}$ would be a very welcome ingredient in our program. Otherwise, MSSM contributions to FCNC and CP violation processes will be relatively small corrections to the SM predictions and high precision measurements will be required.

So, it goes without saying that the experimental determination of these observables is very challenging, requiring sustained hard work along the next years in different experiments, both direct production of SUSY particles at colliders and indirect searches at FCNC and CP violation experiments. In our exposition, all the CP-conserving quantities such as masses and moduli of the different mixing angles are supposed to be known, accurately enough, in order to perform the analysis of the CP-violating quantities. In this scenario we can use direct measurements at high energy colliders, such as the LHC, the ILC, etc, and measurements at FCNC experiments to extract the relevant phases. Nevertheless in this framework our program can only be realized through a long and iterative process with a synergetic high energy-FCNC interplay, in which the first steps will not produce very precise results (see reference [9] for an example of a realistic analysis of flavour independent phases at colliders).

Finally, we would like to relate our expressions with weak basis invariants to the usual

computations in supersymmetric models, both in the mass insertion formalism [43] and in the exact mass eigenstate formalism working with flavour changing vertices. The mass insertion formalism is just a series expansion on the small off-diagonal elements of the squark mass matrices, useful without performing a full diagonalization of them. In first place, we must take into account that, as shown in Appendix B, our invariants can contain an arbitrary function of the internal hermitian mass matrices. These arbitrary functions are, in the Feynman diagram computations, the usual loop functions. Therefore, all we have to do to relate our invariant formalism with the usual Feynman diagram calculations is to express the internal hermitian matrices in terms of projectors, which give us the mixing matrices entering in the process, and combine the mass eigenstates of these matrices in the corresponding loop functions. The squark mass eigenstates are a mixture of left and right-handed squarks; nevertheless it is still possible to express M_Q^2 as a linear combination of the 6 squark masses and the 6×6 squark mixing matrices [25]. Naturally, gauge couplings do not enter in our invariants, but at least we can identify the gauge couplings associated with gauginos and W bosons, as explained in our rules to build flavour diagrams given above. The translation to the mass insertion formalism is also straightforward from here. In this case, we do not express the internal squark mass matrix in terms of projectors, and replace the mass eigenstates by a universal squark mass in the loop functions. Then the hermitian squark mass matrix in the invariant plays the role of a new off-diagonal flavour coupling and the usual mass insertion corresponds directly to $(\delta_L^x)_{ij} = (\mathcal{U}_L^{x\dagger} H_{\tilde{Q}} \mathcal{U}_L^x)_{ij} / m_Q^2$. Notice that, as pointed out in [16], the full invariant must contain additional mixing matrices to be completely weak basis invariant.

4. Conclusions

In this work we have presented the complete machinery necessary to find all the independent WBIs in any extension of the Standard Model and to relate them to physical observables. We have defined weak basis and rephasing invariance, and shown how any flavour process in the Standard Model, and in particular any CP-violating process, can be easily expressed in terms of WBIs. We have introduced a graphical representation of these WBIs as a simple extension of the usual Feynman diagrams. As a practical application, we have found all the independent observables in a reduced version of the MSSM with only three flavour matrices. In this model, we have been able to define a basis of four complex invariants spanning all the observable phases in the model. Then we have chosen four different physical processes to fix these four invariants completely; from there, assuming we know the sparticles masses and moduli of the mixings, we are able to make predictions on any other CP-violating observable in the model.

This formalism can be applied to more complete models, as for instance the full MSSM or any other extension of the SM with new flavour structures. This analysis will be presented in a future work [25].

Acknowledgements

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A. Traces and mixings

In this appendix we prove that any complex invariant in our reduced MSSM can be written as a linear combination of only four independent complex invariants. Using Eq. (3.4) we can concentrate on invariants built only with projectors.

It is obvious that invariants with two projectors are always real and, in fact, Eq. (2.18), they simply carry the moduli of elements of mixing matrices. We diagonalize the hermitian matrices, $H_u = \mathcal{U}_L^{u\dagger} \text{Diag}(m_{u_i}^2) \mathcal{U}_L^u$, $H_d = \mathcal{U}_L^{d\dagger} \text{Diag}(m_{d_i}^2) \mathcal{U}_L^d$ and $H_{\tilde{Q}} = \mathcal{U}^Q \text{Diag}(m_{\tilde{Q}_i}^2) \mathcal{U}^{Q\dagger}$ and define the mixing matrices $V \equiv \mathcal{U}_L^{u\dagger} \mathcal{U}_L^d$ (just the CKM matrix) and $U \equiv \mathcal{U}^{Q\dagger} \mathcal{U}_L^d$. Then, we obtain the invariant moduli

$$\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \right) = |V_{ij}|^2 \quad ; \quad \text{Tr} \left(\mathbf{P}_k^Q \mathbf{P}_j^{d_L} \right) = |U_{kj}|^2. \quad (\text{A.1})$$

The next step is to consider invariants involving three different projectors, $\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \right)$, which can have non-zero imaginary parts. Nevertheless, as $\mathbf{1} = \sum \text{Projectors}$, we can write

$$\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \right) = \sum_{\ell} \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_{\ell}^{d_L} \right) \quad (\text{A.2})$$

Any trace over 3 projectors can be expressed in terms of a sum of traces over 4 projectors where one kind of projector appears twice.

Now, we have two kinds of invariants with 4 projectors. First, invariants that involve the three sorts of projectors $\mathbf{P}_i^{d_L}, \mathbf{P}_i^{u_L}, \mathbf{P}_i^Q$:

$$\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_{\ell}^{d_L} \right), \quad \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^{u_L} \mathbf{P}_{\ell}^Q \right), \quad \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^Q \mathbf{P}_k^{d_L} \mathbf{P}_{\ell}^Q \right), \quad (\text{A.3})$$

then, invariants that involve only two sorts of projectors:

$$\text{Tr} \left(\mathbf{P}_{i_1}^{u_L} \mathbf{P}_{j_1}^{d_L} \mathbf{P}_{i_2}^{u_L} \mathbf{P}_{j_2}^{d_L} \right), \quad \text{Tr} \left(\mathbf{P}_{i_1}^{d_L} \mathbf{P}_{j_1}^Q \mathbf{P}_{i_2}^{d_L} \mathbf{P}_{j_2}^Q \right), \quad \text{Tr} \left(\mathbf{P}_{i_1}^{u_L} \mathbf{P}_{j_1}^Q \mathbf{P}_{i_2}^{u_L} \mathbf{P}_{j_2}^Q \right). \quad (\text{A.4})$$

As in the SM case, using unitarity of the mixing matrix, each family of invariants in Eq. (A.4) provides a single imaginary part.

Using the mixing matrices defined above, we define,

$$J_{i_1 j_1, i_2 j_2}^{(V)} \equiv \text{Tr} \left(\mathbf{P}_{i_1}^{u_L} \mathbf{P}_{j_1}^{d_L} \mathbf{P}_{i_2}^{u_L} \mathbf{P}_{j_2}^{d_L} \right) = V_{i_1 j_1} V_{i_2 j_2}^* V_{i_2 j_2} V_{i_1 j_2}^* \quad (\text{A.5})$$

$$J_{k_1 j_1, k_2 j_2}^{(U)} \equiv \text{Tr} \left(\mathbf{P}_{k_1}^Q \mathbf{P}_{j_1}^{d_L} \mathbf{P}_{k_2}^Q \mathbf{P}_{j_2}^{d_L} \right) = U_{k_1 j_1} U_{k_2 j_2}^* U_{k_2 j_2} U_{k_1 j_2}^* \quad (\text{A.6})$$

$$I_{ij, k\ell} \equiv \text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_{\ell}^{d_L} \right) = V_{ij} V_{i\ell}^* U_{k\ell} U_{kj}^*. \quad (\text{A.7})$$

And now, with $\mathbf{1} = \sum \text{Projectors}$ and $\text{Tr} (A P_i B P_i) = \text{Tr} (A P_i) \text{Tr} (B P_i)$ (for any pro-

jector P_i), we have:

$$\begin{aligned}
\text{Tr} \left(P_{i_1}^{u_L} P_{j_1}^{d_L} P_{i_2}^{u_L} P_k^Q \right) &= \sum_{j_2} \text{Tr} \left(P_{i_1}^{u_L} P_{j_1}^{d_L} P_{i_2}^{u_L} P_k^Q P_{j_2}^{d_L} \right) \\
&= \sum_{j_2} \frac{\text{Tr} \left(P_{i_1}^{u_L} P_{j_1}^{d_L} P_{i_2}^{u_L} P_k^Q P_{j_2}^{d_L} \right) \text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} \right)}{\text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} \right)} \\
&= \sum_{j_2} \frac{\text{Tr} \left(P_{i_2}^{u_L} P_k^Q P_{j_2}^{d_L} \right) \text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} P_{i_1}^{u_L} P_{j_1}^{d_L} \right)}{\text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} \right)} \\
&= \sum_{j_2, j_3} \frac{\text{Tr} \left(P_{i_2}^{u_L} P_{j_3}^{d_L} P_k^Q P_{j_2}^{d_L} \right) \text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} P_{i_1}^{u_L} P_{j_1}^{d_L} \right)}{\text{Tr} \left(P_{i_2}^{u_L} P_{j_2}^{d_L} \right)} \\
&= \sum_{j_2, j_3} \frac{I_{i_2 j_3, k j_2} J_{i_2 j_2, i_1 j_1}^{(V)}}{|V_{i_2 j_2}|^2}. \tag{A.8}
\end{aligned}$$

In a similar way, we obtain

$$\text{Tr} \left(P_i^{u_L} P_{k_1}^Q P_{j_1}^{d_L} P_{k_2}^Q \right) = \sum_{j_2, j_3} \frac{I_{i j_2, k_1 j_3} J_{k_1 j_1, k_1 j_3}^{(U)}}{|U_{k_1 j_3}|^2} \tag{A.9}$$

$$\text{Tr} \left(P_{k_1}^Q P_{i_1}^{u_L} P_{k_2}^Q P_{i_2}^{u_L} \right) = \sum_{j_1, j_2, j_3, j_4} \frac{I_{i_1 j_1, k_1 j_2} I_{i_2 j_1, k_2 j_3} I_{i_2 j_4, k_1 j_1}}{|U_{k_1 j_1}|^2 |V_{i_2 j_1}|^2}. \tag{A.10}$$

Using these relations, the invariant traces in Eqs. (A.4) and (A.3) can be reduced to three different families, $\text{Tr} \left(P_{i_1}^{u_L} P_{j_1}^{d_L} P_{i_2}^{u_L} P_{j_2}^{d_L} \right)$, $\text{Tr} \left(P_{i_1}^{d_L} P_{j_1}^Q P_{i_2}^{d_L} P_{j_2}^Q \right)$ (each one providing a single imaginary part) and $\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right)$. Higher-order invariants (with more than 4 projectors) are easily reduced to the ones considered through the same method.

With $\text{Tr} \left(P_{i_1}^{u_L} P_{j_1}^{d_L} P_{i_2}^{u_L} P_{j_2}^{d_L} \right)$ and $\text{Tr} \left(P_{i_1}^{d_L} P_{j_1}^Q P_{i_2}^{d_L} P_{j_2}^Q \right)$ providing two independent imaginary parts, we turn our attention to $\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right)$, particularly to the number of independent imaginary parts in this family of invariants. The interesting relations now are:

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right) = \frac{\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_m^{u_L} P_\ell^{d_L} \right) \text{Tr} \left(P_m^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right)}{\text{Tr} \left(P_m^{u_L} P_\ell^{d_L} \right) \text{Tr} \left(P_m^{u_L} P_j^{d_L} \right)} \tag{A.11}$$

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right) = \frac{\text{Tr} \left(P_k^Q P_\ell^{d_L} P_m^Q P_j^{d_L} \right) \text{Tr} \left(P_i^{u_L} P_j^{d_L} P_m^Q P_\ell^{d_L} \right)}{\text{Tr} \left(P_m^Q P_\ell^{d_L} \right) \text{Tr} \left(P_m^Q P_j^{d_L} \right)} \tag{A.12}$$

$$\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_\ell^{d_L} \right) = \frac{\text{Tr} \left(P_i^{u_L} P_j^{d_L} P_k^Q P_a^{d_L} \right) \text{Tr} \left(P_i^{u_L} P_a^{d_L} P_k^Q P_\ell^{d_L} \right)}{\text{Tr} \left(P_i^{u_L} P_a^{d_L} \right) \text{Tr} \left(P_k^Q P_a^{d_L} \right)}. \tag{A.13}$$

Equation (A.11) allows us to select an arbitrary i in $\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_\ell^{d_L} \right)$; the second relation, Eq. (A.12), allows us to select an arbitrary k in $\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_\ell^{d_L} \right)$. As the exchange $j \rightleftharpoons \ell$ amounts to a conjugation, that is $\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_\ell^{d_L} \right) = \left[\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_\ell^{d_L} \mathbf{P}_k^Q \mathbf{P}_j^{d_L} \right) \right]^*$, only 3 different $\text{Tr} \left(\mathbf{P}_i^{u_L} \mathbf{P}_j^{d_L} \mathbf{P}_k^Q \mathbf{P}_\ell^{d_L} \right)$ are independent, the ones in which $j \neq \ell$. This number is further reduced to 2 by the third relation, Eq. (A.13); these two invariants, together with $\text{Tr} \left(\mathbf{P}_{i_1}^{u_L} \mathbf{P}_{j_1}^{d_L} \mathbf{P}_{i_2}^{u_L} \mathbf{P}_{j_2}^{d_L} \right)$ and $\text{Tr} \left(\mathbf{P}_{i_1}^{d_L} \mathbf{P}_{j_1}^Q \mathbf{P}_{i_2}^{d_L} \mathbf{P}_{j_2}^Q \right)$, span the 4 observable CP-violating phases that appear in this restricted MSSM.

B. $Z \rightarrow b\bar{s}$ example

As a simple illustrating example we will analyse the CP-violating rate asymmetry $\Gamma(Z \rightarrow b\bar{s}) - \Gamma(Z \rightarrow \bar{b}s)$; let us consider the simplest complex invariant traces that may appear in this observable. As we fix b and s external quarks this requires the presence of \mathbf{P}_3^{dL} and \mathbf{P}_2^{dL} ; there are no other projectors. With two down quark projectors the trace requires two additional matrices to exhibit an imaginary part; as the available matrices are H_u and M_Q^2 , we can expect the presence of the structures

$$\begin{aligned} & \text{Tr} \left(\mathbf{P}_2^{dL} f_1(H_u) \mathbf{P}_3^{dL} f_2(H_u) \right) \quad ; \quad \text{Tr} \left(\mathbf{P}_2^{dL} g_1(M_Q^2) \mathbf{P}_3^{dL} g_2(M_Q^2) \right) \\ & \text{Tr} \left(\mathbf{P}_2^{dL} f_3(H_u) \mathbf{P}_3^{dL} g_3(M_Q^2) \right) \quad ; \quad \text{Tr} \left(\mathbf{P}_2^{dL} g_4(M_Q^2) \mathbf{P}_3^{dL} f_4(H_u) \right), \end{aligned} \quad (\text{B.1})$$

where $f_i(H_u)$ and $g_i(M_Q^2)$ are functions of H_u and M_Q^2 (loop functions).

Let us consider the leading amplitude $A \equiv A(Z \rightarrow b\bar{s})$:

$$A = Z \sim\!\!\sim \begin{array}{c} \nearrow b \\ \searrow \bar{s} \end{array} \begin{array}{c} u_i \\ W \end{array} + Z \sim\!\!\sim \begin{array}{c} \nearrow b \\ \searrow \bar{s} \end{array} \begin{array}{c} \tilde{Q}_i \\ \chi \end{array} \quad (\text{B.2})$$

The first kind of contribution is the SM one; the second one is the first SUSY contribution in our simple MSSM, with squarks and gauginos running in the loop. Schematically:

$$\begin{aligned} |A|^2 = & Z \sim\!\!\sim \begin{array}{c} \circ \\ b \end{array} \begin{array}{c} \curvearrowright \\ u_i \\ W \\ u_j \end{array} \begin{array}{c} \circ \\ \bar{s} \end{array} Z \sim\!\!\sim + Z \sim\!\!\sim \begin{array}{c} \circ \\ b \end{array} \begin{array}{c} \curvearrowright \\ \tilde{Q}_i \\ \chi \\ \tilde{Q}_j \end{array} \begin{array}{c} \circ \\ \bar{s} \end{array} Z \sim\!\!\sim + \\ & 2\text{Re} \left[Z \sim\!\!\sim \begin{array}{c} \circ \\ b \end{array} \begin{array}{c} \curvearrowright \\ u_i \\ W \\ \tilde{Q}_j \end{array} \begin{array}{c} \circ \\ \bar{s} \end{array} Z \sim\!\!\sim \right] \end{aligned} \quad (\text{B.3})$$

Notice that the insertion of 'o' in the diagrams recalls the fact that the b and \bar{s} are external

states. In terms of invariant traces,

$$\begin{array}{c} b \\ \circ \\ \curvearrowright \\ W \\ \curvearrowleft \\ \circ \\ \bar{s} \end{array} \quad \begin{array}{c} Z \text{ wavy} \\ u_i \\ \text{wavy} \\ u_j \\ \text{wavy} \\ Z \end{array} \quad \rightarrow \quad \begin{array}{c} \circ \\ \text{P}_3^{d_L} \\ \text{F}(H_u) \quad \text{F}^\dagger(H_u) \\ \text{P}_2^{d_L} \\ \circ \end{array} = \text{Tr} \left(\text{P}_2^{d_L} F(H_u) \text{P}_3^{d_L} F^\dagger(H_u) \right) \quad (\text{B.4})$$

$$\begin{array}{c} b \\ \circ \\ \curvearrowright \\ Z \sim \text{wavy} \tilde{Q}_i \quad \chi \quad \tilde{Q}_j \text{wavy} Z \\ \curvearrowleft \\ \circ \\ \bar{s} \end{array} \quad \rightarrow \quad \begin{array}{c} \circ \\ \text{P}_3^{d_L} \\ \times \quad \times \quad \times \quad \times \\ G(M_Q^2) \quad G^\dagger(M_Q^2) \\ \times \quad \times \quad \times \quad \times \\ \circ \\ \text{P}_2^{d_L} \end{array} = \text{Tr} \left(\text{P}_2^{d_L} G(M_Q^2) \text{P}_3^{d_L} G^\dagger(M_Q^2) \right) \quad (\text{B.5})$$

$$\begin{array}{c}
b \\
\circ \\
\curvearrowright \\
Z \text{ --- } wavy \text{ --- } u_i \text{ --- } \text{wavy} \text{ --- } \chi \text{ --- } \tilde{Q}_j \text{ --- } wavy \text{ --- } Z \\
\curvearrowleft \\
\bar{s} \\
\circ
\end{array}
\quad \rightarrow \quad
\begin{array}{c}
\circ \\
P_3^{dL} \\
F(H_u) \quad G^\dagger(M_Q^2) \\
\circ \\
P_2^{dL} \\
\circ
\end{array}
= \text{Tr} \left(P_2^{dL} F(H_u) P_3^{dL} G^\dagger(M_Q^2) \right)
\quad (B.6)$$

That is

$$|A|^2 = \text{Tr} \left(\mathbf{P}_2^{d_L} F(H_u) \mathbf{P}_3^{d_L} F^\dagger(H_u) \right) + \text{Tr} \left(\mathbf{P}_2^{d_L} G(M_Q^2) \mathbf{P}_3^{d_L} G^\dagger(M_Q^2) \right) + 2\text{Re} \left[\text{Tr} \left(\mathbf{P}_2^{d_L} F(H_u) \mathbf{P}_3^{d_L} G^\dagger(M_Q^2) \right) \right] \quad (\text{B.7})$$

Similarly the amplitude $\bar{A} = A(Z \rightarrow \bar{b}s)$ is

$$|\bar{A}|^2 = \text{Tr} \left(\mathbf{P}_2^{d_L} F^\dagger(H_u) \mathbf{P}_3^{d_L} F(H_u) \right) + \text{Tr} \left(\mathbf{P}_2^{d_L} G^\dagger(M_Q^2) \mathbf{P}_3^{d_L} G(M_Q^2) \right) + 2\text{Re} \left[\text{Tr} \left(\mathbf{P}_2^{d_L} F^\dagger(H_u) \mathbf{P}_3^{d_L} G(M_Q^2) \right) \right]. \quad (\text{B.8})$$

Decomposing the loop functions in dispersive and absorptive pieces,

$$F(H_u) = F_{Dis}(H_u) + iF_{Abs}(H_u) \quad ; \quad G(M_Q^2) = G_{Dis}(M_Q^2) + iG_{Abs}(M_Q^2), \quad (\text{B.9})$$

we can simplify the CP asymmetry $A_{\text{CP}} = |A|^2 - |\bar{A}|^2$:

$$\begin{aligned}
|A|^2 - |\bar{A}|^2 = & 4\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} F_{Dis}(H_u) \mathbf{P}_3^{dL} F_{Abs}(H_u) \right) \right] + 4\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} G_{Dis}(M_Q^2) \mathbf{P}_3^{dL} G_{Abs}(M_Q^2) \right) \right] + \\
& 4\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} F_{Dis}(H_u) \mathbf{P}_3^{dL} G_{Abs}(M_Q^2) \right) \right] - 4\text{Im} \left[\text{Tr} \left(\mathbf{P}_2^{dL} F_{Abs}(H_u) \mathbf{P}_3^{dL} G_{Dis}(M_Q^2) \right) \right]. \quad (\text{B.10})
\end{aligned}$$

By expanding the different functions, for example $F_{Dis}(H_u) = \sum_j F_{Dis}(m_{u_j}^2) P_j^{u_L}$, we can write

$$\begin{aligned}
|A|^2 - |\bar{A}|^2 = & \\
& 4 \sum_{i,j} F_{Dis}(m_{u_i}^2) F_{Abs}(m_{u_j}^2) \text{Im} \left[\text{Tr} \left(P_2^{d_L} P_i^{u_L} P_3^{d_L} P_j^{u_L} \right) \right] + \\
& 4 \sum_{i,j} G_{Dis}(m_{Q_i}^2) G_{Abs}(m_{Q_j}^2) \text{Im} \left[\text{Tr} \left(P_2^{d_L} P_i^Q P_3^{d_L} P_j^Q \right) \right] + \\
& 4 \sum_{i,j} \left[F_{Dis}(m_{u_i}^2) G_{Abs}(m_{Q_j}^2) - F_{Abs}(m_{u_i}^2) G_{Dis}(m_{Q_j}^2) \right] \text{Im} \left[\text{Tr} \left(P_2^{d_L} P_i^{u_L} P_3^{d_L} P_j^Q \right) \right]. \quad (\text{B.11})
\end{aligned}$$

The asymmetry is thus easily written in terms of different irreducibly complex invariants; Eq. (B.11) is further reduced as there are no absorptive parts in the loops containing squarks or top quarks: $F_{Abs}(m_t^2) = G_{Abs}(m_{Q_i}^2) = 0$.

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